EFFECTIVE CONDUCTIVITY OF SYSTEMS WITH INTERPENETRATING COMPONENTS

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Various methods of calculating effective conductivities of systems with interpenetrating components are compared, and the accuracy of the recommended method is examined.

It is proposed in [1, 2] to determine the effective thermal and electrical conductivities of a two-component heterogeneous system by taking as a model of the system two isotropic interpenetrating space lattices having cubic symmetry. Figure 1A shows an eighth of a unit cell of the model with interpenetrating components, any face of which can be oriented perpendicular to the temperature gradient. In this case the two outside faces of the cell perpendicular to the gradient are isopotential planes and the other four planes are not crossed by the flow lines.

The effective thermal conductivity λ is determined from the equation

$$\langle \vec{j} \rangle = -\lambda \langle \vec{\nabla T} \rangle . \tag{1}$$

For local values the relation

$$\vec{j}_i(\vec{r}) = -\lambda_i \vec{\nabla T}_i(\vec{r}),\ \operatorname{div} \vec{j}_i(\vec{r}) = 0 \tag{2}$$

must be satisfied.

The solution of Eq. (2) together with the boundary conditions leads to cumbersome expressions for the temperature distribution in a unit cell. Therefore, λ was determined in [1, 2] by the approximate Rayleigh method [3], which consists of subdividing inhomogeneous composite bodies by auxiliary infinitely thin isopotential planes and planes which are not crossed by the flow lines. In this case finding the effective thermal conductivity of a heterogeneous system reduces to the determination of the effective conductivity of an equivalent network of thermal resistances obtained by subdividing a unit cell.

In [1], son Frey proposed determining the approximate value of λ by drawing an isopotential plane $a-a$ perpendicular to the average heat flux. Then the equivalent thermal network takes the form of Fig. 1B. On the network diagram R_i denotes the thermal resistance of the i-th cube of Fig. 1A:

$$R_1 = 1/\lambda_1(L-\Delta),\ R_2 = (L-\Delta)/\Delta^2\lambda_1,\ R_{2'} = 1/\lambda_1\Delta,$$
$$R_3 = 1/\lambda_2\Delta,\ R_4 = \Delta/\lambda_2(L-\Delta)^2,\ R_{4'} = 1/\lambda_2(L-\Delta). \tag{3}$$

The effective thermal resistance R = 1/λL is determined from the network of Fig. 1B. After transformations the following expression is obtained for the effective transfer coefficient [1, 2]:

$$\lambda_F = \lambda_1 \left[\frac{1-C}{C^2+\nu(1-C^2)} + \frac{C}{C(2-C)+\nu(1-C)^2} \right]^{-1},\ \nu = \frac{\lambda_2}{\lambda_1}. \tag{4}$$

The value of C = Δ/L is uniquely related to the m_2 volume concentration of the second component [1, 2]:

$$2C^3 - 3C^2 + 1 = m_2. \tag{5}$$

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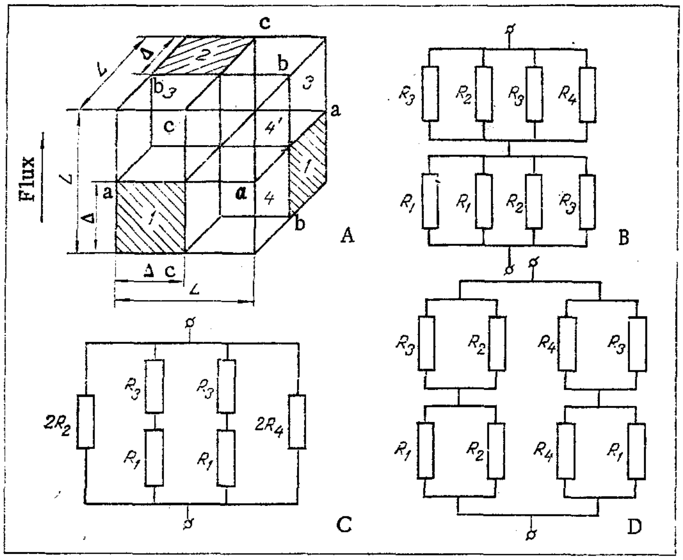

Fig. 1. Calculation of effective conductivity: A) unit cell of model with interpenetrating components. Equivalent networks of unit cell: B) isothermal subdivision by plane $a-a$; C) adiabatic subdivision by planes b—b and c—c; D) combination subdivision.

Independently of [1], it was proposed in [4] to determine the approximate value of λ by drawing adiabatic planes b—b and c—c. The corresponding equivalent thermal network is shown in Fig. 1C, and the expression for the transfer coefficient takes the form [2, 4]

$$\lambda_D = \lambda_1\{C^2 + \nu(1-C)^2 + 2\nu C(1-C)[\nu C + 1 - C]^{-1}\}. \tag{6}$$

The difference between Eqs. (4) and (6) reaches a maximum of 45% for $\nu = 0$ and decreases as ν increases from 0 to 1.

The comparison performed in [2] of the approximate values of λ_F and λ_D with a similar curve for $\lambda_e = f(\lambda_1, \lambda_2, m_2)$ with $\nu = 0$ obtained by a numerical method is shown in Fig. 2, from which it follows that

$$\lambda_D < \lambda_e < \lambda_F . \tag{7}$$

Therefore, as an improved approximation of λ it was proposed [2] to take

$$\lambda = \frac{1}{2}(\lambda_D + \lambda_F), \tag{8}$$

which gives values very close to those obtained by numerical calculation and preserves the property of invariance with respect to the interchange of subscripts of the components.

We note that an adiabatic—isothermal subdivision of a unit cell is also possible by taking c—c as a plane which the flow lines do not cross and $a-a$ as isothermal. Then the corresponding equivalent thermal network takes the form shown in Fig. 1D. Using the same procedure we obtain for the effective conductivity

$$\lambda_c = \lambda_1\left[\frac{C^2 + \nu(1-C)C}{\nu C(1-C) + (1-C+C^2)} + \nu\,\frac{C(1-C) + \nu(1-C)^2}{C(1-C) + \nu(1-C+C^2)}\right]. \tag{9}$$

A comparison of the approximate values λ_F, λ_D, and λ_c with λ_e calculated numerically is shown in Fig. 2. It is clear from the figure that λ_c differs from λ_e by 6.6% at most. Hence we can conclude that an approximate calculation of the effective conductivity of heterogeneous systems and composite bodies using the Rayleigh method of sections with a combination subdivision gives a λ which differs from the value calculated numerically by much less than 10% and can be used to obtain analytic relations if the solution of the differential equations for these systems presents serious mathematical difficulties.

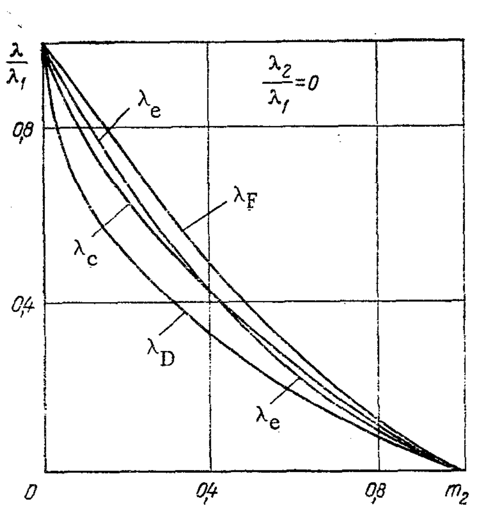

Fig. 2. Effective conductivity of a system with interpenetrating components as a function of concentration.

Instead of the widely used Eqs. (6) and (8) for calculating the conductivity of heterogeneous systems with interpenetrating components Eq. (9) should be used.

NOTATION

$\langle \vec{j} \rangle$, average heat flux; $\langle \nabla \vec{T} \rangle$, average temperature gradient; $\vec{j}_i(\vec{r})$, local heat flux in i-th component; λ_i, thermal conductivity of i-th component; λ, effective thermal conductivity; $\nabla \vec{T}_i(\vec{r})$, local temperature gradient in i-th component; $C = \Delta/L$, relative size of block of unit cell; λ_F, λ_D, λ_e, λ_c, effective thermal conductivity calculated by the son Frey and Dul'nev equations, obtained by a numerical method (exact value), and by the combination equation (9).

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SOME PROBLEMS IN MEASURING THERMAL CONDUCTIVITY USING THE COAXIAL CYLINDERS METHOD. II

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The results of an investigation of the thermal conductivity of five liquids using an apparatus based on the absolute coaxial cylinders method are presented. The measurements are made using three different measuring gaps. Procedural problems in measuring thermal conductivity are considered.

To investigate the thermal conductivity we used equipment based on the absolute coaxial cylinders method [1]. In the present paper we give the results of an experimental study of the thermal conductivity of water, toluene, n-heptane, and n-propyl and isopropyl alcohols in the temperature range from 20°C to 140°C. The measurements were made for three different thicknesses of the layer of material being investigated in order to clarify how the value of

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